Answer Key

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1. Simplify each expression, and eliminate any negative exponent(s).

Rules:

$a^n \cdot a^m = a^{n+m}$

$\frac{b^n}{b^m} = b^{n-m}$

$(c^n)^m = c^{n \cdot m}$

$d^{-n} = \frac{1}{d^{+n}}$

a. $(5x^{-3}y^3)(7x^2)^2 = \left(\frac{5y^3}{x^3}\right)(7^2\ x^{2\cdot 2}) = \frac{5y^3}{x^3}(49x^4)$

$= 5 \cdot 49 \cdot \frac{x^4}{x^3} \cdot y^3 = \boxed{245xy^3}$

b. $\frac{y^{-2}z^{-3}}{y^{-1}} = \frac{y^1}{y^2\ z^3}$

$= \frac{1}{y^{2-1} z^3} = \boxed{\frac{1}{yz^3}}$

c. $\left(\frac{a^3b^{-2}}{b^3}\right)^2 = \left(\frac{a^3}{b^3 \cdot b^2}\right)^2 = \left(\frac{a^3}{b^{3+2}}\right)^2 = \left(\frac{a^3}{b^5}\right)^2$

$= \frac{a^{3\cdot 2}}{b^{5\cdot 2}}$

$= \boxed{\frac{a^6}{b^{10}}}$

Rule

$\sqrt[a]{x^a} = x$

unless

$\sqrt[\text{even index}]{x^{\text{even \#}}}$

$= |x^{\text{odd \#}}|$

2. Simplify the expression. Assume that *a* and *b* denote any real numbers. (Assume that *a* denotes a positive number.)

$\sqrt[4]{80a^7b^4} = \sqrt[4]{2^4 \cdot 5\, a^{4+3}\, b^4} = \sqrt[4]{2^4 \cdot 5 a^4 a^3 b^4}$

$= \sqrt[4]{2^4}\ \sqrt[4]{5}\ \sqrt[4]{a^4}\ \sqrt[4]{a^3}\ \sqrt[4]{b^4}$

$= 2\sqrt[4]{5} \cdot |a| \cdot \sqrt[4]{a^3} \cdot |b|$

$= \boxed{2|ab|\sqrt[4]{5a^3}}$

80
2 40
2 20
2 10
2 5

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Rule:
$a(b+c)$
$= ab + ac$

3. Find the sum, difference, or product. (Simplify your answer completely.)

$7(x^2 - 3x + 5) - 6(x^2 - 2x + 1)$

$7(x^2) + 7(-3x) + 7(5) - 6(x^2) - 6(-2x) - 6(1)$

$7x^2 - 21x + \boxed{35} - 6x^2 + 12x - 6$

$7x^2 - 6x^2 \quad - 21x + 12x \quad + 35 - 6$

$\boxed{x^2 - 9x + 29}$

4. Factor the difference of squares.

Rule $A^2 - B^2 = (A+B)(A-B)$

$49a^2 - 4$

$(7a)^2 - (2)^2$

$(7a+2)(7a-2)$

5. Factor the trinomial.

AC method : $ax^2 + bx + c$

Find ac $= 7(5) = 35$

What factors of 35 add to -36

$-1 \quad -35$

Replace bx term with new factors and factor by grouping.

$7x^2 - 36x + 5$

$7x^2 - 35x - 1x + 5$

$(7x^2 - 35x) + (-1x + 5)$

$7x(x-5) + -1(x-5)$

$\boxed{(x-5)(7x-1)}$

6. Factor the trinomial.

$x^2 + 10x - 39$

$\boxed{(x-3)(x+13)}$

What factors of -39 will add to 10

-39

$1 \quad 39$

$3 \quad 13 \rightarrow +3 - 13 \neq 10$

$-3 + 13 = 10$

Note: Can use ac method if you choose to.

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7. Perform the multiplication or division and simplify.

$$\frac{x^2+3x+2}{x^2+9x+20}\cdot\frac{x^2+7x+10}{x^2+4x+4}$$

Factor each numerator and denominator

$$=\frac{(x+2)(x+1)}{(x+5)(x+4)}\cdot\frac{(x+2)(x+5)}{(x+2)(x+2)}$$

Remember: $\frac{a}{b}\cdot\frac{c}{d}=\frac{ac}{bd}$

and $\frac{a}{\cancel{b}}\cdot\frac{\cancel{b}}{d}=\frac{a}{d}$

$$=\frac{\cancel{(x+2)}(x+1)\cancel{(x+2)}\cancel{(x+5)}}{\cancel{(x+5)}(x+4)\cancel{(x+2)}\cancel{(x+2)}}$$

$$=\boxed{\frac{x+1}{x+4}}$$

8. Perform the addition or subtraction and simplify.

$$\frac{1}{x+6}+\frac{3}{x-1}$$

LCD: $(x+6)(x-1)$

Rule: $\frac{a}{b}+\frac{c}{d}=\frac{ad+cb}{bd}$

$$=\frac{1}{x+6}\cdot\frac{(x-1)}{(x-1)}+\frac{3}{(x-1)}\cdot\frac{(x+6)}{(x+6)}$$

1st Find LCD
rewrite each fraction with LCD
add fractions.

$$=\frac{1(x-1)+3(x+6)}{(x+6)(x-1)}$$

$$=\frac{x-1+3x+18}{(x+6)(x-1)}=\frac{x+3x-1+18}{(x+6)(x-1)}=\boxed{\frac{4x+17}{(x+6)(x-1)}}$$

9. The given equation is either linear or equivalent to a linear equation. Solve the equation.

$$7(1-x)=8(1+2x)+9$$

Distribute
combine like terms
Solve for x.

$$7-7x=8+16x+9$$

$$7-7x=17+16x$$

$$+7x \qquad +7x$$

$$7=17+23x$$

$$-17 \qquad -17$$

$$\frac{-10}{23}=\frac{23x}{23}$$

$$\frac{-10}{23}=x$$

$$\boxed{x=-10/23}$$

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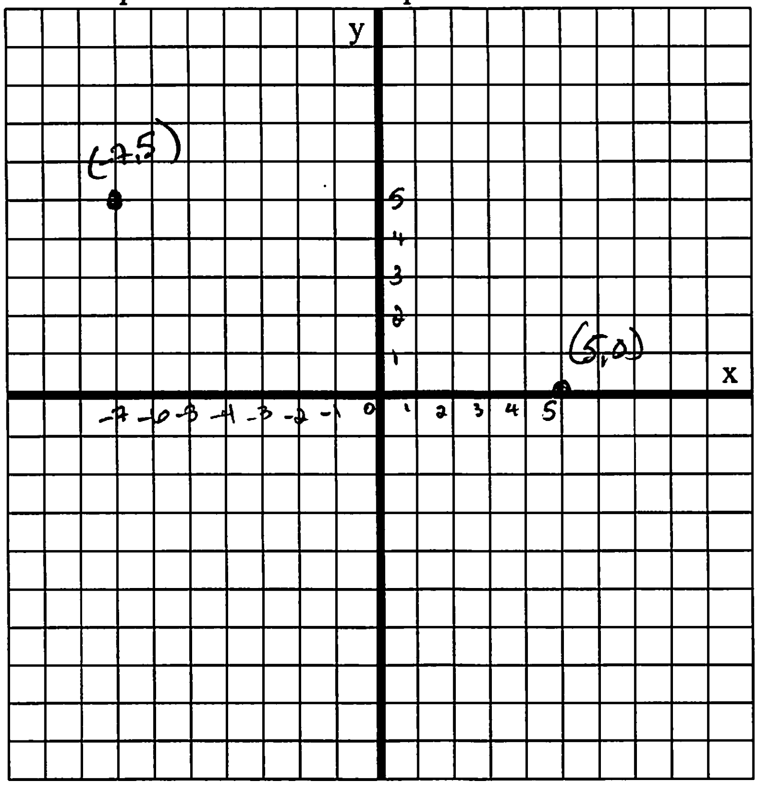
10. A pair of points is given. $(-7,5),(5,0)$

a. Plot the points in a coordinate plane.



$(-7,5) = (x_1, y_1)$
$(5,0) = (x_2, y_2)$

$d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

$M = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

b. Find the distance between them.

$d = \sqrt{(5-(-7))^2 + (0-5)^2} = \sqrt{(5+7)^2+(-5)^2} = \sqrt{(12)^2+(-5)^2}$
$= \sqrt{144+25} = \sqrt{169} = \boxed{13}$

c. Find the midpoint of the segment that joins them.

$(x,y) = \left(\frac{-7+5}{2}, \frac{5+0}{2}\right) = \left(\frac{-2}{2}, \frac{5}{2}\right) = \boxed{\left(-1, 5/2\right)}$

11. Find the x- and y-intercepts of the graph of the equation. (If answer does not exist, enter DNE.)

$5x - 6y = 120$

$(x,0)$ Let $y=0$ → X-intercept $= \boxed{(24,0)}$

$(0,y)$ Let $x=0$ → Y-intercept $\boxed{(0,-20)}$

x-intercept

$5x - 6(0) = 120$
$\frac{5x}{5} = \frac{120}{5}$
$x = 24$
$(24,0)$

y-intercept

$5(0) - 6y = 120$
$\frac{-6y}{-6} = \frac{120}{-6}$
$y = -20$
$(0,-20)$

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12. Find the slope of the line through P and Q.

$P(5,-5), Q(8,-1)$

$(5,-5) = (x_1, y_1)$
$(8,-1) = (x_2, y_2)$

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$m = \frac{-1-(-5)}{8-5}$
$= \frac{-1+5}{8-5}$
$= \boxed{4/3}$

13. Find the equation of the line that satisfies the given conditions.

Through $(-1,-2)$ and $(6,5)$.

$(-1,-2) = (x_1, y_1)$
$(6,5) = (x_2, y_2)$

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$y - y_1 = m(x - x_1)$
$y = mx + b$

$m = \frac{5-(-2)}{6-(-1)} = \frac{5+2}{6+1} = \frac{7}{7} = 1$

use $m = 1$ $(-1,-2) = (x_1, y_1)$

$y - y_1 = m(x - x_1)$
$y - (-2) = 1(x - (-1))$
$y + 2 = 1(x+1)$
$y + 2 = x + 1$
$-2 \quad -2$
$\boxed{y = x - 1}$

14. Find all real solution of the equation by factoring. (Enter your answer as a comma-separated list.)

$x^2 - 10x + 24 = 0$

x = $\boxed{4, 6}$

what multiplies to -10 but adds to 24?

$x^2 - 10x + 24 = 0$
$(x-6)(x-4) = 0$

$x - 6 = 0$
$+6 \quad +6$
$x = 6$

$x - 4 = 0$
$+4 \quad +4$
$x = 4$

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15. Find all real solutions of the equation. (Enter your answers as a comma-separated list. If there is no real solution, enter NO REAL SOLUTION.)

$x^2 - 10x + 1 = 0$

x = $\boxed{5-4\sqrt{6},\ 5+4\sqrt{6}}$

Since no factors of 1 add to 10...
Use the quadratic formula

$ax^2+bx+c=0$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$x^2-10x+1=0$

$a=1 \quad b=-10 \quad c=1$

$x = \frac{-(-10) \pm \sqrt{(-10)^2-4(1)(1)}}{2(1)}$

$= \frac{10 \pm \sqrt{100-4}}{2}$

$= \frac{10 \pm \sqrt{96}}{2} = \frac{10 \pm \sqrt{16 \cdot 6}}{2} = \frac{10}{2} \pm \frac{4\sqrt{6}}{2} = 5 \pm 4\sqrt{6}$

$= 5-4\sqrt{2},\ 5+4\sqrt{6}$

16. Evaluate the product, and write the results in the form $a + bi$.

$(9 - i)(7 + 5i)$

Remember that $\sqrt{-1} = i$
so $i^2 = -1$

$9(7+5i) - i(7+5i)$

$63 + 45i - 7i - 5i^2$

$63 + 38i - 5(-1)$

$63 + 38i + 5 = \boxed{68 + 38i}$

17. Find all real solutions of the equation. (Enter your answers as a comma-separated list.)

$x^3 = 25x$

$\boxed{x = -5, 0, 5}$

1st equation = 0
2nd factor and solve for x.

$x^3 = 25x$
$-25x \quad -25x$

$x^3 - 25x = 0$

Remember
$A^2 - B^2 = (A+B)(A-B)$

$x(x^2-25) = 0$

$x(x+5)(x-5) = 0$

$x = 0 \qquad \begin{array}{r} x+5=0 \\ -5\ \ -5 \\ \hline x=-5 \end{array} \qquad \begin{array}{r} x-5=0 \\ +5\ \ +5 \\ \hline x=+5 \end{array}$

$\boxed{x = -5, 0, 5}$

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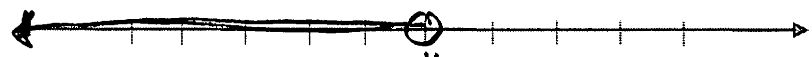
18. Solve the linear inequality. Express the solution using interval notation.

$2 - 3x > 3$

Remember when multiplying or dividing by a negative, the inequality sign switches.

Graph the solution set.



$$2 - 3x > 3$$
$$-2 \qquad -2$$
$$-3x > 1$$
$$\frac{-3x}{-3} \quad \frac{1}{-3}$$
$$x > -1/3$$

Interval notation
$(-\infty, -1/3)$

19. Solve the equation. (Enter your answers as a comma-separated list. If there is no solution, enter NO SOLUTION.)

$3|x + 6| + 4 = 19$

$$-4 \quad -4$$
$$\frac{3|x+6|}{3} = \frac{15}{3}$$
$$|x+6| = 5$$

When working with absolute values remember

$$|a| = \begin{cases} a & \text{if } a \geq 0 \\ -a & \text{if } a < 0 \end{cases}$$

1st isolate the absolute value term.

If $x+6 \geq 0$ then

$$x+6 = 5$$
$$-6 \quad -6$$
$$\boxed{x = -1}$$

If $x+6 < 0$ then

$$x+6 = -5$$
$$-6 \quad -6$$
$$\boxed{x = -11}$$

$$\boxed{x = -11, -1}$$

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20. Hooke's Law states that the force needed to keep spring stretched x units beyond its natural length is directly proportional to x. Here the constant of proportionality is called the **spring constant**.

a. Write Hooke's Law as an equation. (Use k for the constant of proportionality.)

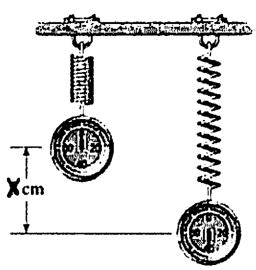
Look at: "the force is directly proportional to x." to write the equation:

$$\text{Force} = kx \quad \text{or}$$

$$\boxed{F = kx}$$

b. If the spring has a natural length of 6 cm and a force of 35 N is required to maintain the spring stretched to a length of 10 cm, find the spring constant.

$k = \boxed{8.75}$



Force = 35 N

X = stretched length (of the spring) − natural length (of the spring)

$X = 10\text{cm} - 6\text{cm}$

$X = 4\text{ cm}$

$\text{Force} = kx$

$\frac{35N}{4} = \frac{k(4\text{ cm})}{4}$

$k = 35/4 = 8.75\ N/\text{cm}$

c. What force is needed to keep the spring stretched to a length of 14 cm?

Find X first: X = stretched length − natural length

$X = 14\text{cm} - 6\text{cm} = 8\text{ cm}$

$F = kx$

$F = (8.75\ N/\text{cm})(8\text{ cm})$

$F = \boxed{70\ N}$

21. Find the domain of the function. (Enter your answer using interval notation.)

$$f(x) = \frac{x^4}{x^2+x-6}$$

Domain rule for fractions: $\frac{1}{\text{term}}$, term $\neq 0$

$x^2+x-6 \neq 0$ Factor

$(x+3)(x-2) \neq 0$

$x+3 \neq 0$, $-3\ -3$, $x \neq -3$

$x-2 \neq 0$, $+2\ +2$, $x \neq 2$

Everything but $x \neq -3, 2$

graph: $-\infty$ … -3 … 2 … ∞

$$\boxed{(-\infty,-3)\cup(-3,2)\cup(2,\infty)}$$

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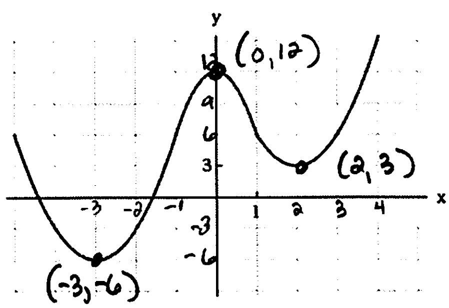
22. Complete the table.

$g(x) = |8x + 7|$

x	$g(x)$
-3	$\vert 8(-3)+7\vert = \boxed{17}$
-2	$\vert 8(-2)+7\vert = \boxed{9}$
0	$\vert 8(0)+7\vert = \boxed{7}$
1	$\vert 8(1)+7\vert = \boxed{15}$
3	$\vert 8(3)+7\vert = \boxed{31}$

$|8(-3)+7| = |-24+7| = |-17| = 17$
$|8(-2)+7| = |-16+7| = |-9| = 9$
$|8(0)+7| = |0+7| = |7| = 7$
$|8(1)+7| = |8+7| = |15| = 15$
$|8(3)+7| = |24+7| = |31| = 31$

23. The graph of a function is given. Use the graph to estimate the following.



a. All the local maximum and minimum values of the function and the values of x at which each occurs

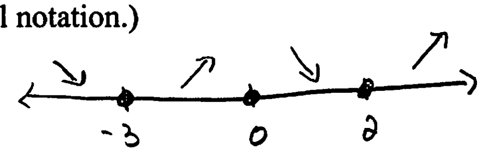
Local Maximum: $(x, y) = \boxed{(0, 12)}$

Local Minimum: $(x, y) = \boxed{(-3, -6)}$

Local Minimum: $(x, y) = \boxed{(2, 3)}$

b. The interval on which the function is increasing and on which the function is decreasing. (Enter your answer using interval notation.)

Increasing: $\boxed{[-3, 0] \cup [2, \infty)}$



Decreasing: $\boxed{(-\infty, -3] \cup [0, 2]}$

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24. A function f is given, and the indicated transformations are applied to its graph (in the given order). Write the equation for the final transformed graph.

$f(x) = x^2$; stretched vertically by a factor of 2, shift downward 8 units, and shift 9 units to the right.

$y = \boxed{2(x-9)^2 - 8}$

$\underline{\text{stretch factor}}\ (x\ \underline{\overset{+\text{ left}}{-\text{ right}}})^2\ \underline{\overset{+\text{ up}}{-\text{ down}}}$

remember the function

$\_\ (x\ \_)^2\ \_\_$

25. Use $f(x) = 4x - 5$ and $g(x) = 2 - x^2$ to evaluate the expression.

a. $(f \circ g)(x) = f(g(x)) = f(2-x^2) = 4(2-x^2) - 5$
$= 8 - 4x^2 - 5$
$= \boxed{3 - 4x^2}$

b. $(g \circ f)(x) = g(f(x)) = g(4x-5) = 2 - (4x-5)^2$
$= 2 - (4x-5)(4x-5)$
$= 2 - (16x^2 - 20x - 20x + 25)$
$= 2 - 16x^2 + 20x + 20x - 25$
$\boxed{= -16x^2 + 40x - 23}$

26. Find the function f whose graph is a parabola with the given vertex and that passes through the given point.

Vertex: $(3, -3)$; point: $(4,2)$

$\boxed{f(x) = 5(x-3)^2 - 3}$

$(3,-3) = (h,k)$ and $(4,2) = (x,y)$

$y = a(x-h)^2 + k$
$2 = a(4-3)^2 - 3$
$2 = a(1) - 3$
$2 = 1a - 3$
$+3 \qquad +3$
$5 = a$
$y = 5(x-3)^2 - 3$

Standard form:
$f(x) = a(x-h)^2 + k$
where (h,k) = vertex
$f(x) = y$.
Find "a" then plug a, (h,k) into the standard form equation.

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27. Find the quotient and remainder using long division.

$$\frac{x^6+4x^4-3x^2-12}{x^2+4}$$

Quotient: $\boxed{x^4-3}$

Remainder $\boxed{0}$

$$\begin{array}{r} x^4 \qquad\qquad\qquad\qquad\qquad\qquad\qquad \\ x^2+0x+4\ \overline{)\ \cancel{x^6}+\cancel{0x^5}+\cancel{4x^4}+0x^3-3x^2+0x-12} \\ -\cancel{x^6}-\cancel{0x^5}-\cancel{4x^4} \qquad\qquad\qquad\qquad\qquad \\ \hline -3\cancel{x^2}+\cancel{0x}-\cancel{12} \\ +3\cancel{x^2}+\cancel{0x}+\cancel{12} \\ \hline 0 \end{array}$$

$x^4(x^2+0x+4)$
$x^6+0x^5+4x^4$
then subtract.
$-3(x^2+0x+4)$
$-3x^2+0x-12$
then subtract.

28. Find all the zeros of the polynomial. (Enter your answer as a comma-separated list. Enter all answers including repetitions.)

$P(x) = x^3 + 5x^2 + 4x + 20$

$\boxed{x = -5,\ 2i,\ -2i}$

Factor by grouping
set each factor = 0.
solve for x.

$\underbrace{x^3+5x^2}+\underbrace{4x+20}$

$x^2(x+5)+4(x+5)$

$(x+5)(x^2+4)$

$x+5=0$
$-5\ -5$
$x=-5$

$x^2+4=0$
$-4\ -4$
$x^2=-4$
$\sqrt{x^2}=\sqrt{-4}$
$x=\pm\sqrt{-1\cdot4}$
$x=\pm 2i$

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29. Find the intercepts and asymptotes. (If an answer does not exist, enter DNA. Enter your asymptotes as a comma-separated list of equations if necessary.)

$$s(x) = \frac{(4x-12)}{(x-4)(x+1)}$$

numerator = 0 — X-intercept: $(x, y) = \boxed{(3,0)}$

plug zero in for all x's — Y-intercept: $(x, y) = \boxed{(0,3)}$

Denominator = 0 — Vertical asymptote(s): $\boxed{x=4,\ x=-1}$

3 rules: $\frac{ax^n}{bx^m}$ $\begin{cases} n<m & y=0 \\ n=m & y=a/b \\ n>m & \text{None} \end{cases}$ — Horizontal asymptote: $\boxed{y=0}$

X intercept

$4x-12=0$
$+12 \quad +12$
$4x = 12$
$\frac{4x}{4} = \frac{12}{4}$
$x = 3$

y intercept

$y = \frac{4(0)-12}{(0-4)(0+1)} = \frac{-12}{(-4)(1)} = 3$

vertical

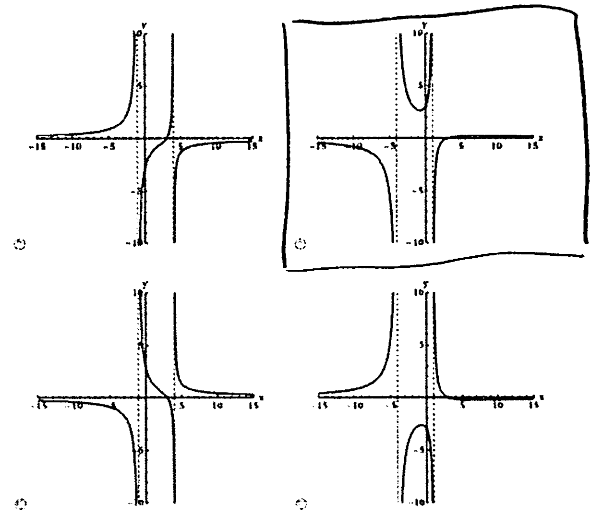
$x-4=0$, $x=4$; $x+1=0$, $x=-1$

horizontal

$\frac{4x-12}{(x-4)(x+1)} = \frac{4x-12}{x^2-4x+x-4}$

$n=1$
$m=2$

Sketch the graph of the rational function.



ans

State the domain and range. Use a graphing device to confirm your answer. (Enter your answer using interval notation.)

look at vertical asymptotes — Domain: $\boxed{(-\infty,-1)\cup(-1,4)\cup(4,\infty)}$

Range: $\boxed{(-\infty,\infty)}$

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30. Use the elimination method to find all solutions of the system of equations.

$$\begin{cases} 3x + 5y = 28 \\ 6x + \ \ y = 11 \end{cases}$$

$(x, y) = \boxed{(1,5)}$

Elimination method:
Want one of the unknown values, x or y. to cancel when equations are added together.

$3x+5y=28$
$6x+y=11$

if you want to cancel the x terms, I could:

$+6(3x+5y=28) \Rightarrow \quad \cancel{18x} + 30y = 168$
$-3(6x+y=11) \quad\quad\ \ -\cancel{18x} - 3y = -33$

$$\frac{27y}{27} = \frac{135}{27}$$

$y = 5$

$3x+5y = 28$
$3x+5(5) = 28$
$3x + 25 = 28$
$\quad\ -25 \ \ -25$

$$\frac{3x}{3} = \frac{3}{3}$$

$x = 1$

There are easier ways to find the answer, this is just the way I saw first.